

# Energy Dependence of High Moments for Net-proton Distributions

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**Abstract.** High moments of multiplicity distributions of conserved quantities are predicted to be sensitive to critical fluctuations. To understand the effect of the non-critical physics backgrounds on the proposed observable, we have studied various moments of net-proton distributions with AMPT, Hijing, Therminator and UrQMD models, in which no QCD critical point physics is implemented. It is found that the centrality evolution of various moments of net-proton distributions can be uniformly described by a superposition of emission sources. In addition, in the absence of critical phenomena, some moment products of net-proton distributions, related to the baryon number susceptibilities in Lattice QCD calculations, are predicted to be constant as a function of the collision centrality. We argue that a non-monotonic dependence of the moment products as a function of the beam energy may be used to locate the QCD critical point.

## 1. Introduction

Heavy-ion reactions at high energy allow us to study the QCD phase diagram experimentally [1]. At vanishing baryon chemical potential ( $\mu_B = 0$ ), Lattice QCD calculations predict that a cross-over from the hadronic phase to the Quark Gluon Plasma (QGP) phase will occur above a critical temperature. The temperature range for the cross-over has been estimated to be about 170 - 190 MeV [2]. QCD based model calculations indicate that at large  $\mu_B$  the transition from the hadronic phase to the QGP phase could be of first order with a critical point at the boundary to the cross-over, the QCD Critical Point (QCP) [3]. The location of the QCP or even its existence are not confirmed [4]. The possibility of the existence of the QCP has motivated our interest to search for it with the RHIC beam energy scan program [5]. By decreasing the collision energy down to a center of mass energy of 5 GeV we will be able to vary the baryo-chemical potential from  $\mu_B \sim 0$  to  $\mu_B$  of about 500 MeV.

A characteristic feature of a critical point is the increase and divergence of the correlation length ( $\xi$ ) and of critical fluctuations. In heavy-ion reactions, finite size

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effects, rapid expansion, and critical slowing down could wash out those effects. For example, the critical correlation length in heavy-ion collisions is expected to be about 2-3 fm [6]. A clear signature of a critical point in an energy scan would be non-monotonic behavior of non-gaussian multiplicity fluctuations.

Recently, theoretical calculations have shown that high moments of multiplicity distributions of conserved quantities, such as net-baryon, net-charge, and net-strangeness, are sensitive to the correlation length  $\xi$  [7].

In Lattice QCD calculation with  $\mu_B = 0$ , higher order susceptibilities of the baryon number, which can be related to the higher order moments of the net-baryon multiplicity distributions, show a non-monotonic behavior near  $T_c$  [8]. A similar behavior is expected for the finite  $\mu_B$  region. Experimentally, it is hard to measure the net-baryon number while the net-proton number is measurable. Theoretical calculations show that fluctuations of the net-proton number can be used to infer the net-baryon number fluctuations at the QCP [9].

In this paper we study the energy dependence of net-proton multiplicity distributions for several models in terms of Skewness (  $S$  ) and Kurtosis (  $\kappa$  ). This is a feasibility study for the future data analysis from the energy scan at RHIC.

## 2. Observables

We introduce various moment definitions of the event-by-event multiplicity distributions: Mean,  $M = \langle N \rangle$ , Variance,  $\sigma^2 = \langle (\Delta N)^2 \rangle$ , Skewness,  $S = \langle (\Delta N)^3 \rangle / \sigma^3$ , and Kurtosis,  $\kappa = \langle (\Delta N)^4 \rangle / \sigma^4 - 3$ , where  $\Delta N = N - \langle N \rangle$ . Skewness and Kurtosis are used to characterize the asymmetry and peakness of the multiplicity distributions, respectively. They are also used to demonstrate the non-Gaussian fluctuation feature near the QCP, in particular a sign change of the skewness may be a hint of crossing the phase boundary [7, 10].

To understand the centrality evolution of these moments, we introduce the *Identical Independent Emission Source* (IIES) assumption. Here the colliding system consists of a large number of emission sources and the final multiplicity of particles is the sum of the multiplicities from individual emission sources. The relation between various moments and the number of emission sources for the  $i^{th}$  centrality can be expressed as:

$$(1) : \frac{M_i}{N_i} = \frac{\sum_{i=1}^n M_i}{\sum_{i=1}^n N_i} = M(x), (2) : \frac{\sigma_i^2}{N_i} = \frac{\sum_{i=1}^n \sigma_i^2}{\sum_{i=1}^n N_i} = \sigma^2(x)$$

$$(3) : \frac{N_i}{(1/S_i^2)} = \frac{\sum_{i=1}^n N_i}{\sum_{i=1}^n (1/S_i^2)} = S^2(x), (4) : \frac{N_i}{(1/\kappa_i)} = \frac{\sum_{i=1}^n N_i}{\sum_{i=1}^n (1/\kappa_i)} = \kappa(x)$$

where  $M_i, \sigma_i, S_i, \kappa_i$  ( $i = 1, 2, \dots, n$ ) are the moments extracted from the multiplicity distribution of the  $i^{th}$  centrality and  $N_i$  is the corresponding number of emission sources.

$M(x), \sigma(x), S(x), \kappa(x)$  are various moments of the multiplicity distributions for each emission source. From Eqs.(1)-(4), we obtain:

$$(5) : \frac{M_i}{\sum_{i=1}^n M_i} = \frac{\sigma_i^2}{\sum_{i=1}^n \sigma_i^2} = \frac{1/S_i^2}{\sum_{i=1}^n (1/S_i^2)} = \frac{1/\kappa_i}{\sum_{i=1}^n (1/\kappa_i)} = \frac{N_i}{\sum_{i=1}^n N_i}$$

which shows the connection between the emission source distributions and the various moments of multiplicity distributions. To investigate the centrality evolution of those moments, we fit the normalized mean value  $M_i / \sum_{i=1}^n M_i$  with a function  $f(< N_{part} >)$ , where  $< N_{part} >$  is the average number of participants. Then, we obtain:

$$(6) : M(< N_{part} >) = \left( \sum_{i=1}^n M_i \right) * f(< N_{part} >)$$

$$(7) : \sigma(< N_{part} >) = \sqrt{\left( \sum_{i=1}^n \sigma_i^2 \right) * f(< N_{part} >)}$$

$$(8) : S(< N_{part} >) = 1 / \sqrt{\left( \sum_{i=1}^n 1/S_i^2 \right) * f(< N_{part} >)}$$

$$(9) : \kappa(< N_{part} >) = 1 / \left[ \left( \sum_{i=1}^n 1/\kappa_i \right) * f(< N_{part} >) \right]$$

Consequently, the centrality evolution of various moments can be uniformly described by the function  $f(< N_{part} >)$ . From those equations it also follows that the moment products,  $S\sigma$ ,  $\kappa\sigma/S$  and  $\kappa\sigma^2$ , are constant as a function of  $< N_{part} >$ .

### 3. Results

We calculated the various moments of net-proton ( $\Delta p = N_p - N_{\bar{p}}$ ) distributions from transport models (AMPT [11], Hijing [12], UrQMD [13]) and a thermal model (Therminator [14]). By using several models with different physics implemented, we can study the effects of physics correlations and backgrounds that are trivially present in the data and that might modify purely statistical emission patterns, like resonance decays, jet-production (Hijing), coalescence mechanism of particle production (AMPT), thermal particle production (Therminator), and hadronic rescattering (AMPT, UrQMD).

The kinetic coverage of protons and anti-protons used in our analysis is  $0.4 < p_T < 0.8$  GeV/c and  $|y| < 0.5$ . In Fig.1, typical net-proton distributions for three centralities, 0–5%, 30–40%, and 70–80%, of Au+Au collisions at  $\sqrt{s_{NN}} = 200$  GeV are calculated with the UrQMD model. The shapes of net-proton distributions are different for the three centralities. For the most central collisions, 0–5%, the net-proton distribution is wider compared to more peripheral collisions. The discrepancies in shapes will be reflected in the values of the different moments.

Fig. 2 shows the  $< N_{part} >$  dependence of four moments ( $M, \sigma, S, \kappa$ ) extracted from net-proton distributions of Au+Au collisions at  $\sqrt{s_{NN}} = 200$  GeV for the various

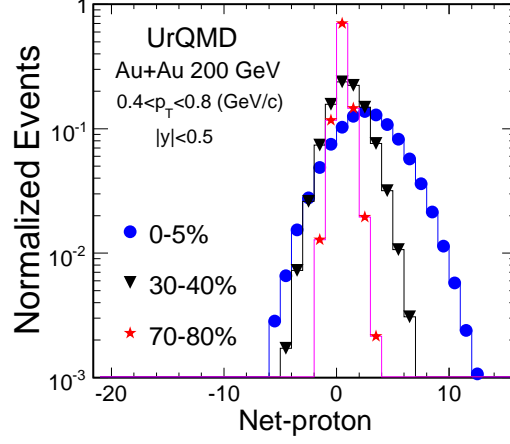


Figure 1: Typical event by event net-proton multiplicity distributions of various centralities for Au+Au collisions at  $\sqrt{s_{NN}} = 200$  GeV calculated by UrQMD model.

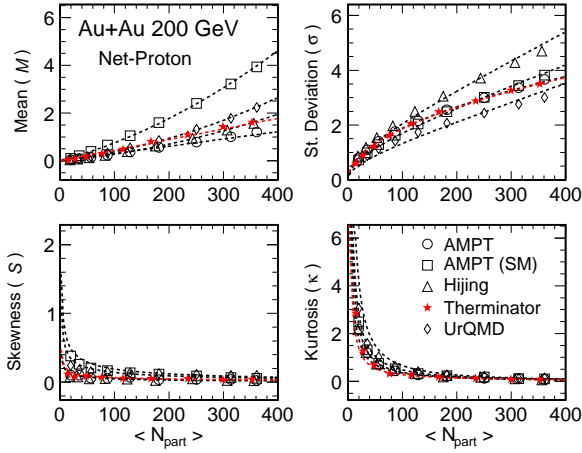


Figure 2: Centrality dependence of various moments of  $\Delta p$  distributions for Au+Au collisions at  $\sqrt{s_{NN}} = 200$  GeV from various models. The dashed lines represent the expectations for statistical emission.

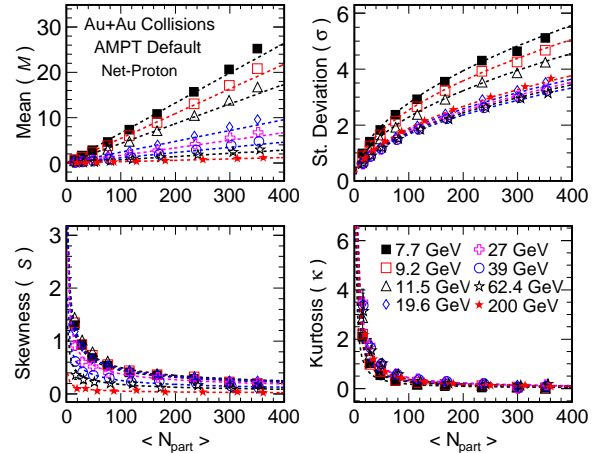


Figure 3: Centrality dependence of various moments of  $\Delta p$  distributions for Au+Au collisions at various energies from the AMPT model. The dashed lines represent the expectations for statistical emission.

models.  $M$  and  $\sigma$  show a monotonic increase with  $\langle N_{part} \rangle$  for all of the models, while  $S$  and  $\kappa$  decrease monotonically. In Fig. 3, we choose the default AMPT model to evaluate the centrality evolution of the various moments of net-proton distributions for various energies.  $M$  shows a linear increase with  $\langle N_{part} \rangle$  and a decrease with  $\sqrt{s_{NN}}$ .  $\sigma$  increases monotonically with  $\langle N_{part} \rangle$  while it has non-monotonic dependence on  $\sqrt{s_{NN}}$ .  $S$  is positive and decreases with increasing  $\langle N_{part} \rangle$  and  $\sqrt{s_{NN}}$ . The net-proton distributions become more symmetric for central collision and higher energies.  $\kappa$  decreases with  $\langle N_{part} \rangle$  and is similar for all energies. The dashed lines in Fig. 2 and Fig. 3 are resulting from Eqs. (6)-(9) to evaluate the centrality evolution of the various moments. To apply our formulas, we fit the normalized mean value in Equ.(5) with the

function  $f(< N_{part} >)$ . For AMPT String Melting (SM) [11], Hijing [12] and UrQMD [13] models, a 2nd order polynomial,  $f(< N_{part} >) = a < N_{part} >^2 + b < N_{part} >$  is applied, while a linear function  $f(< N_{part} >) = a * < N_{part} >$  is employed for AMPT default [11] and Therminator [14] models. Once the function  $f(< N_{part} >)$  is obtained, the centrality evolution of the other moments is completely determined by Eqs. (7)-(9). It is obvious that the centrality evolution of the various moments of net-proton distributions in Fig. 2 and in Fig. 3 can be well described by the dashed lines.

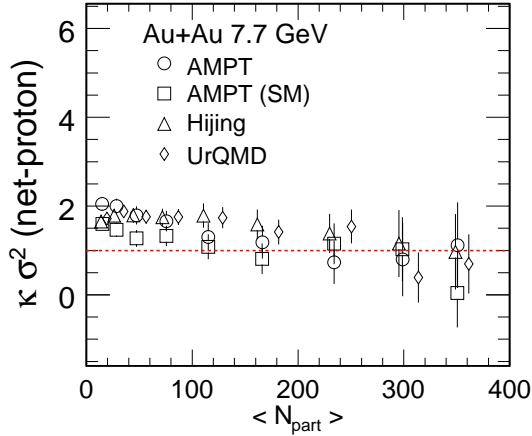


Figure 4: The  $\kappa\sigma^2$  of net-proton distributions for Au+Au 7.7 GeV collisions as a function of  $< N_{part} >$  from various models.

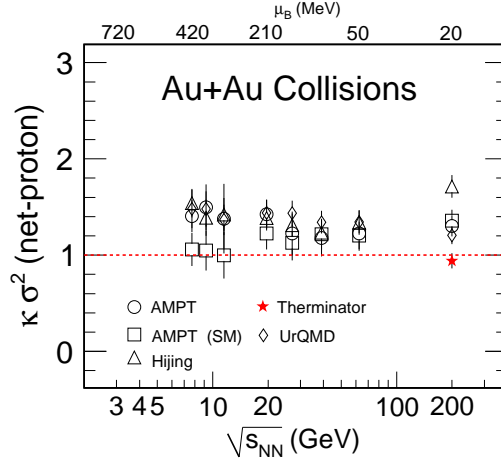


Figure 5: The  $\kappa\sigma^2$  of net-proton distributions in Au+Au collisions as a function of  $\sqrt{s_{NN}}$  for various models.

The  $\kappa\sigma^2$  of the net-proton distributions of Au+Au collisions at  $\sqrt{s_{NN}} = 7.7$  GeV as a function of  $< N_{part} >$  is shown in Fig. 4 for various models.  $\kappa\sigma^2$  is constant with respect to  $< N_{part} >$  within the errors, which is consistent with the expectation from the *IIES* assumption. Fig. 5 shows the energy dependence of the  $\kappa\sigma^2$  of net-proton distributions for various models. The top of the figure shows the  $\mu_B$  value corresponding to the various energies. The values shown are averaged within the centrality range studied. The results from various models show no dependence on energy and are close to unity. This suggests that the  $\kappa\sigma^2$  of net-proton distributions is not affected very much by the non-QCP physics at different beam energies, such as the change of  $\mu_B$  [1], and the collective expansion [15]. Note that the result from the pure thermal model, Therminator, is much closer to unity compared to others. Actually, if proton and anti-proton have independent poisson distributions, the difference of protons and anti-protons should distribute as a Skellam distribution [16], for which  $\kappa\sigma^2$  is unity. A large deviation from constant as a function of  $< N_{part} >$  and collision energy for  $\kappa\sigma^2$  may indicate new physics, such as critical fluctuations.

## 4. Summary and Outlook

Higher moments of the distribution of conserved quantities are predicted to be sensitive to the correlation length at QCP and to be related to the susceptibilities computed in Lattice QCD. Various non-QCP models (AMPT, Hijing, Therminator, UrQMD) have been applied to study the non-QCP physics background effects on the high moments of net-proton distributions. The centrality evolution of the high moments from models can be well described by the scaling derived from the *IIES* assumption and the moment products  $S\sigma$ ,  $\kappa\sigma/S$  and  $\kappa\sigma^2$  of net-proton distributions are constant with respect to  $\langle N_{part} \rangle$ .  $\kappa\sigma^2$  is also found to be constant as a function of energy for various models.

Our model study can serve as a background study of the behavior expected from known physics effects for the RHIC beam energy scan, that will span values of  $\mu_B$  from 100 to about 550 MeV. The presence of a critical point in that region may result in non-gaussian fluctuations and in correlated emission. Then the *IIES* assumption will break down. This is expected to lead to non-monotonic behavior of the observables studied here as a function of collision energy.

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